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On the Grand 5R Five-bar-based Inherently Balanced Linkage Architecture

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Abstract. With the inherent balance approach, balanced mechanisms are synthesized from inherently balanced linkage architectures as a basis which is contrary to aiming at balancing a specific given mechanism. This has the advantage that practical solutions are obtained with few or no countermasses. In this paper the Grand 5R Five-bar-based Inherently Balanced Linkage Architecture is presented which is a new Grand Architecture to be used as a basis in the inherent design approach. Its composition is explained as a combination of the 5 existing principal vector systems with in total 65 links and which is 58 times overconstrained. By eliminating redundant links, numerous normally constrained inherently balanced linkage solutions can be synthesized. An example of a 3-DoF inherently balanced linkage solution is shown together with its inherent balance conditions.

Keywords. Inherent balance, gravity balance, shaking force balance, five-bar linkage, synthesis

1. Introduction

The challenge in designing balanced mechanisms is to find applicable solutions. For high-speed high-precision robotics for instance, where dynamic balance is an important aspect to minimize base vibrations [5,3,1,2,13,6], the common methods to balance multi-degree-of-freedom (multi-DoF) mechanisms suffer of significant additional mass, inertia, and complexity [12]. Also for large moving and deployable structures such as movable roofs and façades, where force balance is important for safe motions requiring minimal effort and energy usage [4,8,11], solutions are required with minimum additional mass and complexity.

The main cause of not finding applicable solutions is that common methods aim at balancing a given mechanism. Since the mechanism is given, the possible balance solutions that can be found are limited and balance can only be obtained by unavoidable additional elements. However, it is more logical to do the contrary: to design balanced mechanisms by basing them on balance properties and to have their geometries and kinematics depend on these balance properties. This approach is known as inherent balancing

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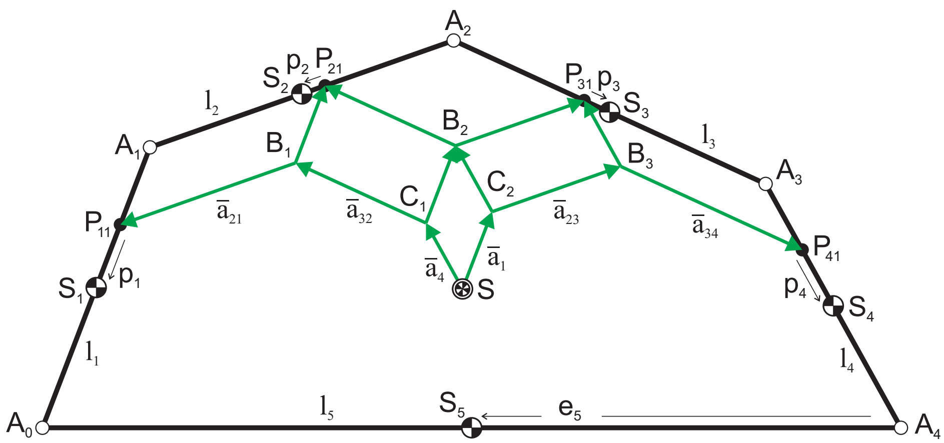

Figure 1. 5R Five-bar linkage $A_0A_1A_2A_3A_4$ with system of principal vectors defining the common CoM in S.

[8,9] and has resulted in a variety of new balanced linkages not needing countermasses or additional elements [10].

For shaking force balance and also for gravity force balance it is important that the center of mass (CoM) of the complete mechanism is in a stationary point in the base link for any of its motions. With the method of principal vectors as fundament, various inherently balanced linkage architectures can be designed which are linkages that are solely based on the essential kinematic relations for balance. Recently an advanced 'Grand' linkage architecture based on the 4R Four-bar linkage with 24 links in total was presented from which 32 inherently balanced linkage solutions were derived [10].

This paper aims at extending the inherent balance theory by presenting an architecture based on the 5R Five-bar linkage, the *Grand 5R Five-bar-based Inherently Balanced Linkage Architecture* including all possible links and interconnections. First the composition of the Grand Architecture is explained and subsequently the use for synthesis is shown by two examples of inherently balanced linkage solutions.

2. Composition of the Grand 5R Five-bar-based Inherently Balanced Linkage Architecture

The common CoM of a 5R Five-bar linkage can be described with respect to its links by a system of principal vectors, which is illustrated in Fig. 1 [8,9]. Five links with a length l_i, a mass m_i, and the link CoM in S_i are connected in a closed loop by revolute pairs in joints A_0, A_1, A_2, A_3, and A_4. The principal vectors $\bar{a}_1$, $\bar{a}_{21}$, $\bar{a}_{23}$, $\bar{a}_{32}$, $\bar{a}_{34}$, and $\bar{a}_4$ describe the locations of the link CoMs with respect to the common CoM by means of the four principal points P_{11}, P_{21}, P_{31}, and P_{41}, forming parallelograms with principal dimensions a_1, a_{21}, a_{23}, a_{32}, a_{34}, and a_4. Distances p_i define the link CoM positions of four links with respect to their principal points and e_5 defines the link CoM S_5 with respect to A_4, as illustrated. The links are mass symmetric with respect to the line through their joints.

Since the principal vectors have constant lengths independent of the pose of the linkage - the principal dimensions are constant parameters - the vectors can be transformed

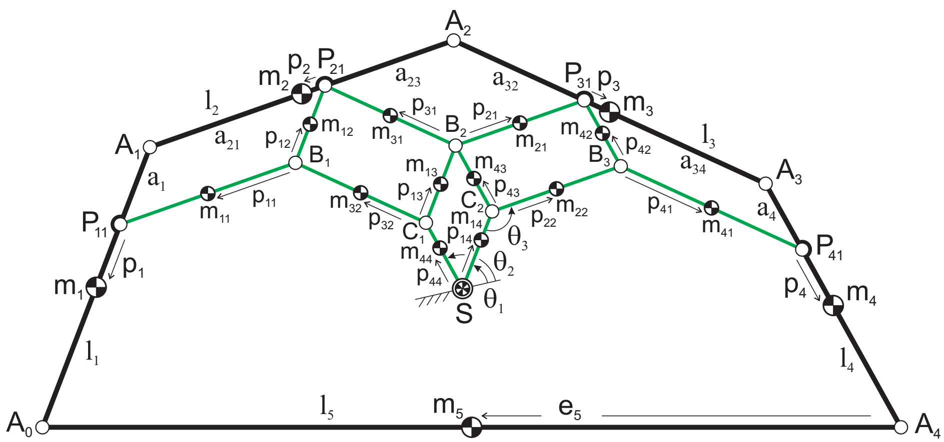

Figure 2. 3-DoF closed-chain principal vector linkage with the common CoM of the 17 links in S for any of its motions. With S as fixed base joint the linkage is force balanced with respect to the base.

into real links with mass. This is shown in Fig. 2 where these 12 principal vector links together with the 5 links of the five-bar are connected with revolute pairs, resulting in a planar linkage of 17 links with 6 parallelograms. This linkage is known as a 3-DoF closed-chain principal vector linkage [8] in which the common CoM of all 17 links is in the joint in S for any possible pose. When S is designed as a fixed base joint, then a force balanced linkage is obtained with the three DoFs θ_1, θ_2, and θ_3. As illustrated, two of the three DoFs can be driven directly from the base, while the third DoF needs to be driven indirectly, for instance from within the linkage.

The mass m_{jk} of each principal vector link jk is located at a distance p_{jk} from one of its two joints, as illustrated. The force balance conditions, which determine the exact mass parameter values for which S is the common CoM of all 17 links, can be derived from [8] as:

$$
\begin{aligned}
m_1 p_1 &= m_{T1} a_1 + m_{12} p_{12} + m_{13} p_{13} + m_{14} p_{14} - m_5^b l_1 \qquad (1)\\
m_2 p_2 &= (m_3 + m_4 + m_5^a + m_{41} + m_{42}) a_{23} + m_{21} p_{21} + m_{22} p_{22} - (m_1 + m_5^b) a_{21} - m_{11} p_{11}\\
m_3 p_3 &= (m_1 + m_2 + m_5^b + m_{11} + m_{12}) a_{32} + m_{31} p_{31} + m_{32} p_{32} - (m_4 + m_5^a) a_{34} - m_{41} p_{41}\\
m_4 p_4 &= m_{T4} a_4 + m_{42} p_{42} + m_{43} p_{43} + m_{44} p_{44} - m_5^a l_4
\end{aligned}
$$

with $m_{T1} = m_2 + m_3 + m_4 + m_5 + m_{21} + m_{22} + m_{31} + m_{41} + m_{42} + m_{43}$, $m_{T4} = m_1 + m_2 + m_3 + m_5 + m_{11} + m_{12} + m_{13} + m_{21} + m_{31} + m_{32}$, $m_5^a = m_5(1 - e_5/l_5)$, and $m_5^b = m_5 e_5/l_5$. One of the results of these conditions is that each link CoM is located halfway its link, which is the natural location for a designed link in practice and reflects the philosophy of inherent balance perfectly.

The principal vector linkage in Fig. 2 is based on a single system of principal vectors. For the five-bar linkage there exist 5 different principal vector systems. In Fig. 3 the linkage of Fig. 2 is shown to which, in orange, principal vector links based on a second system of principal vectors are added with principal points P_{22}, P_{32}, P_{42}, and P_{51}. This set of links is based on the principal vector system directly neighboring the first system in green. It is shown also how the links of both sets interconnect. Joint S here is

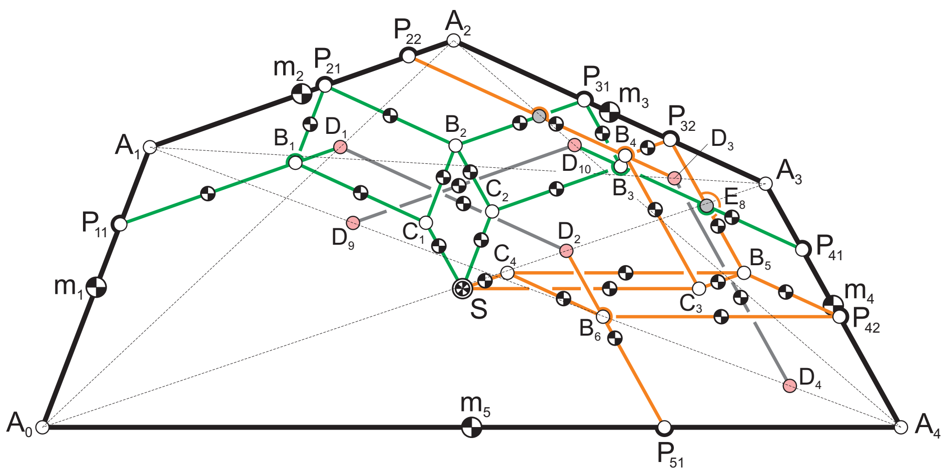

Figure 3. Five-bar linkage with two sets of principal vector links based on two neighboring systems of principal vectors together with their interconnections.

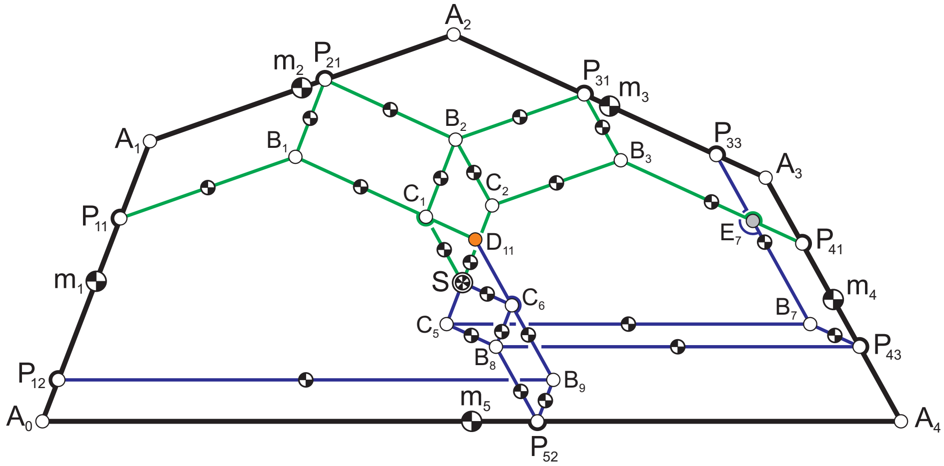

Figure 4. Five-bar linkage with two sets of principal vector links based on two not neighboring systems of principal vectors together with their interconnections.

a joint among 4 different links and joint E_8 can be added since the related links form a parallelogram $P_{32}A_3P_{41}E_8$. Also a link can be added between joints D_1 to D_2, which are joints with the extended links $P_{11}B_1$ and $P_{51}B_6$, respectively. With link D_1D_2 the five-bar linkage $A_0P_{11}D_1D_2P_{51}$ is obtained which is similar of shape to the five-bar $A_0A_1A_2A_3A_4$. This also means that joint D_1 lays on the line through A_0 and A_2 and that D_2 lays on the line through A_0 and A_3.

Because of this combination of two sets of principal vector links, the resulting linkage in Fig. 3 is 5 times kinematically overconstrained: 2 overconstraints of the two extra links in S, 1 overconstraint of additional link D_1D_2, and 2 overconstraints of joint E_8. In

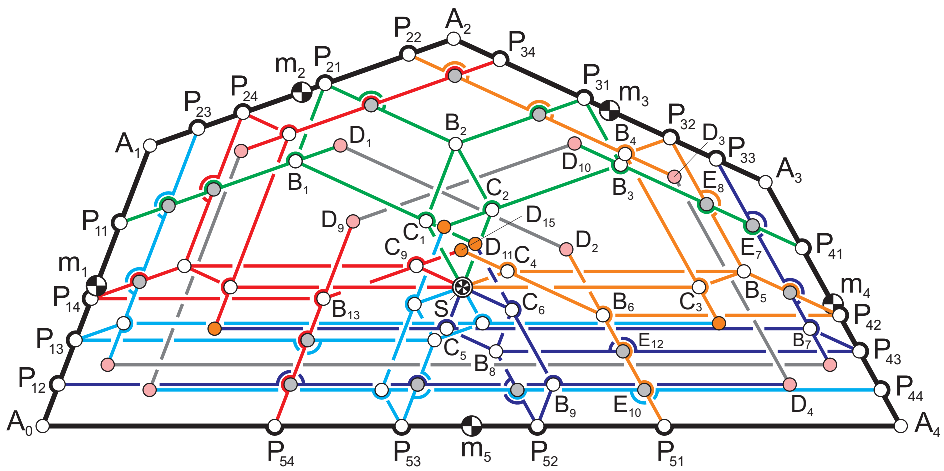

Figure 5. Grand 5R Five-bar-based Inherently Balanced Linkage Architecture with 65 links, 58 overconstraints, and 3-DoF motion with joint S as the common CoM of all links.

general, each set of principal vector links has two of such neighboring sets of principal vector links, one on each side.

In Fig. 4 the linkage of Fig. 2 is shown with, in purple, the set of principal vector links based on a third system of principal vectors not neighboring the first system in green and with principal points P_{33}, P_{43}, P_{52}, and P_{12}. These two sets of links are interconnected by joint E_7, forming parallelogram $P_{33}A_3P_{41}E_7$, and joint D_{11} which is a joint between the extended links B_1C_1 and B_9C_6. Also here in total four links have S as a common joint. Special is that links C_2S and C_5S are aligned and can be merged into a single link C_2C_5 since they move synchronously at all times. In that case the complete linkage in Fig. 4 is movable with 7 overconstraints: 2 of joint E_7, 2 of joint D_{11}, 2 of the two extra links in S, and 1 of having the single rigid link C_2C_5. Also here in general each set of principal vector links has two of such not neighboring sets of principal vector links.

When all 5 sets of principal vector links are combined into a single linkage together with their interconnections, then the result in Fig. 5 is obtained. This linkage with S as the common CoM of all 65 links and with 3-DoF motion capability will be referred to as the *Grand 5R Five-bar-based Inherently Balanced Linkage Architecture*. Each set of principal vector links has its own color. The red set of links is comparable to the orange set of links as explained in Fig. 3 and the blue set of links is comparable to the purple set of links as explained in Fig. 4. As a result, there are in total 5 extra links with the pink colored joints D_{1-10}, in each corner or about each joint A_{0-4} there are three joints E_{1-15} colored grey and forming parallelograms, and about S there are 5 joints D_{11-15} on extended links colored orange. For clarity, not all the joint labels have been depicted in the illustration. Combining all sets of links does not limit the motion capability, the complete linkage architecture remains movable about S with 3-DoF motions. It however is 58 times overconstrained for the case that the 10 links in S have been merged into 5 single links as explained for link C_2SC_5 in Fig. 4. In the next section it is explained how this high number of overconstraints is very useful in the synthesis of normally constrained inherently balanced linkage solutions derived from this Grand Architecture.

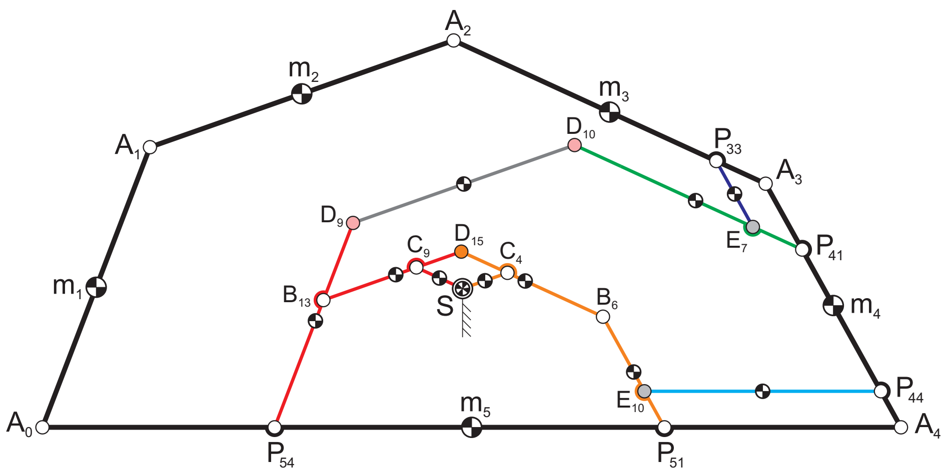

Figure 6. Normally constrained inherently balanced linkage solution where S is the common CoM of all 15 links for any of its 3-DoF motions.

3. Synthesis of Normally Constrained Inherently Balanced Linkage Solutions

Because of the high number of overconstraints, i.e. the high kinematic redundancy, from the Grand Architecture in Fig. 5 a variety of normally constrained inherently balanced linkage solutions can be obtained by simply removing any undesired redundant link, for which there are numerous possibilities. For instance when all internal links are removed but the links $P_{54}D_9$, D_9D_{10}, $D_{10}P_{41}$, $P_{33}E_7$, $B_{13}D_{15}$, $D_{15}B_6$, B_6P_{51}, $E_{10}P_{44}$, C_4S, and C_9S, then the normally constrained linkage solution in Fig. 6 is obtained which has 15 links in total. Here the links $P_{54}D_9$, D_9D_{10}, and $D_{10}P_{41}$ form the five-bar linkage $P_{54}D_9D_{10}P_{41}A_4$ which is similar to $A_0A_1A_2A_3A_4$ and is constrained to move synchronously by means of link $P_{33}E_7$. Also links $P_{54}B_{13}$, $B_{13}D_{15}$, $D_{15}B_6$, B_6P_{51} form a five-bar linkage $P_{54}B_{13}D_{15}B_6P_{51}$ which is similar to $A_0A_1A_2A_3A_4$ and is constrained to move synchronously by means of five-bar linkage $P_{54}D_9D_{10}P_{41}A_4$ and link $P_{44}E_{10}$. This similar five-bar linkage has the two links C_4S and C_9S connected which form a parallelogram with joint S as the common CoM of all the 15 links. With S as a fixed base pivot the complete solution is a 3-DoF inherently balanced linkage.

In the specific composition of Fig. 6 a relation to the work of Shchepetil'nikov [7] can be observed, who found a geometry comparable to the internal links for graphical analysis of the motion of the common CoM of linkages, referring to the links $P_{54}D_9$, D_9D_{10}, and $D_{10}P_{41}$ and to the links $B_{13}D_{15}$, $D_{15}B_6$, B_6P_{51} as 'double contour transformations'.

Another example of a normally constrained inherently balanced linkage solution is shown in Fig. 7, which is obtained by removing all internal links from the Grand Architecture in Fig. 5 besides links $P_{32}B_5$, B_5P_{42}, $P_{42}B_6$, B_6D_{15}, $D_{15}C_9$, C_9C_6, C_6B_8, B_8P_{52}, B_8E_{12}, B_6P_{51}, B_5C_4, and C_4S. It has 17 links in total with S as their common CoM and as fixed base pivot for 3-DoF inherently balanced motions.

At first sight, the linkage in Fig. 7 seems to consist of mainly 5 parallelograms with the parallelogram $C_4D_{15}C_9S$ appearing as a kind of strange add-on. This parallelogram can be redesigned by shifting link $D_{15}C_9$ along link $D_{15}B_6$ such that C_9 coincides with

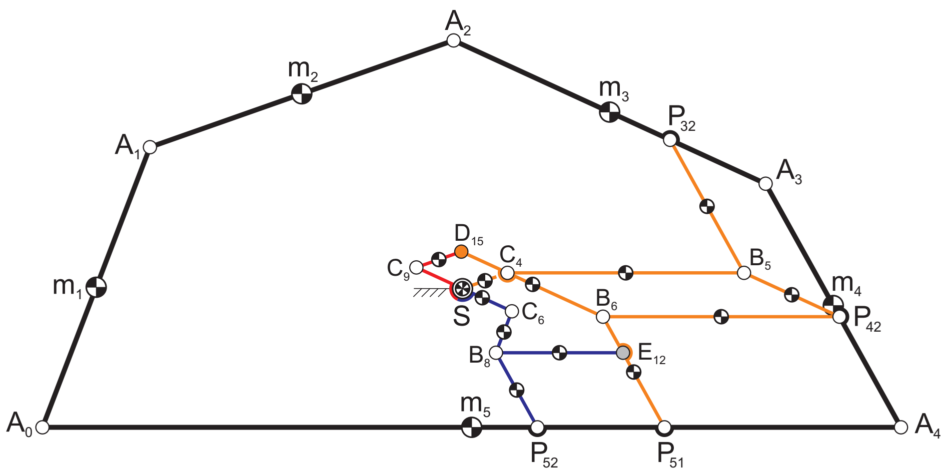

Figure 7. Normally constrained inherently balanced linkage solution where S is the common CoM of all 17 links for any of its 3-DoF motions.

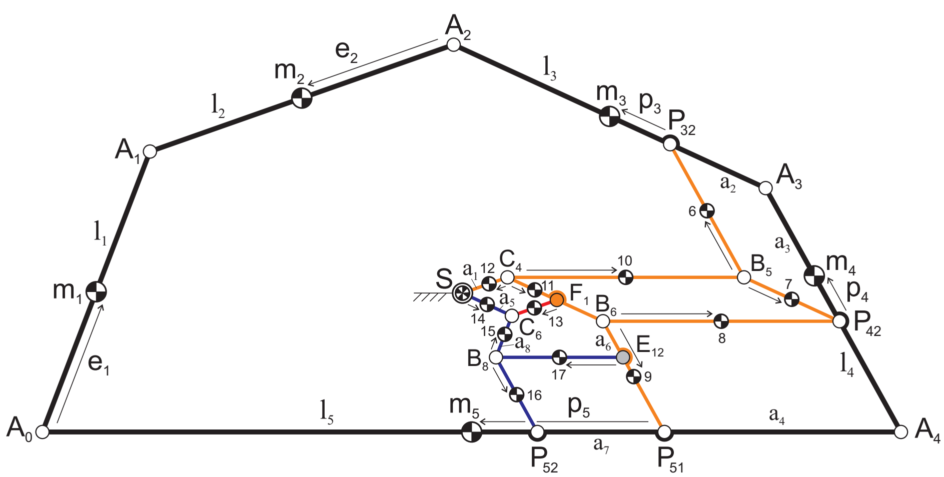

Figure 8. Inherently balanced linkage solution of Fig. 7 with modification for compactness, obtaining a similar five-bar linkage $B_8C_6F_1B_6E_{12}$, with the link mass parameters included.

joint C_6 of which the solution is shown in Fig. 8. This modification does not affect the inherent balance capability of the linkage since it does not affect the relative motions of the links while it results in a more compact design. It is interesting to observe that after this modification the similar five-bar $B_8C_6F_1B_6E_{12}$ is obtained. Different from the Grand Architecture that is based on the 4R Four-bar linkage which includes numerous similar four-bars [10], in the Grand Architecture based on the 5R Five-bar linkage only few similar five-bars are found. However more of them are obtained after modification of an inherently balanced linkage solution.

All internal links, links 6 until 17, have a mass m_i with their link CoM located at a distance p_i from one of the joints as indicated by the arrows, which is along the line through their link joints. The balance conditions of an inherently balanced linkage for which S is the common CoM of all links consist in general of two groups: the kinematic balance conditions and the mass balance conditions. The kinematic balance conditions follow from the similarity properties of the Grand Architecture and determine the links to move synchronously and in parallel. They also define the similarity of the two similar five-bar linkages with:

$$\frac{a_1}{l_2} = \frac{a_2 - a_5}{l_3} = \frac{a_6}{l_4} = \frac{a_7}{l_5} = \frac{a_8}{l_1} \tag{2}$$

The mass balance conditions determine the relations among the link mass values and their centered location in each link. These conditions can be derived with the method presented in [8,10] and result for the linkage in Fig. 8 into:

$$\begin{aligned}
m_2e_2 + m_1^b l_2 + m_{12}p_{12} + m_{13}p_{13} &= (m_{tot} - m_{14} - m_{15}^b)a_1 \\
(m_1^b + m_2)(l_3 - a_2) + m_3p_3 &= (m_1^a + m_4 + m_5 + m_8 + \\
&\quad m_9 + m_{15}^a + m_{16} + m_{17})a_2 + \\
&\quad m_7p_7 + m_{11}p_{11} + (m_{13} + m_{15}^b)a_5 + m_{14}p_{14} \\
(m_1^b + m_2 + m_3)a_3 + m_4p_4 + m_6p_6 &= (m_1^a + m_5)(l_4 - a_3) + m_9p_9 + \\
&\quad (m_{15}^a + m_{16} + m_{17})a_6 + m_{16}p_{16} \\
m_1^a(l_5 - a_4) + m_5p_5 + & \\
(m_{15}^a + m_{16})a_7 + m_{17}p_{17} &= (m_1^b + m_2 + m_3 + m_4 + m_6 + m_7)a_4 + \\
&\quad m_8p_8 + m_{10}p_{10}
\end{aligned} \tag{3}$$

with $m_{tot} = \sum_{i=1}^{17} m_i$, $m_1^a = m_1(1 - e_1/l_1)$, $m_1^b = m_1e_1/l_1$, $m_{15}^a = m_{15}(1 - p_{15}/a_8)$, and $m_{15}^b = m_{15}p_{15}/a_8$. With the four kinematic balance conditions and the four mass balance conditions in total 8 parameters can be calculated, for instance the parameters a_1 to a_8 for which all other values can be freely chosen.

4. Conclusion

In this paper the Grand 5R Five-bar-based Inherently Balanced Linkage Architecture was presented and it was explained how it is composed of the five sets of principal vector links together with their interconnections. The linkage architecture has 3-DoF motion capability with the common CoM of all the 65 links located in a joint of five links. Because of the high number of 58 kinematic overconstraints a variety of normally constrained linkages can be derived by removing undesired redundant links to obtain an inherently balanced linkage solution. This was shown with the synthesis of two examples where for one of them the balance conditions were presented as well.

Within the Grand Architecture multiple similar five-bar linkages can be found, however various of them are only obtained after shifting links within the derived normally constrained balanced linkage solution, as it was shown for the second example. The implications and use of this are interesting for further investigation.

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