Graphic Analysis of the Linear and Angular Momentum of a Dynamically Balanced 1-DoF Pantographic Linkage

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Abstract. This article presents a graphical analysis method for the verification of the gravity force balance and shaking force and shaking moment balance of a 1-DoF pantographic linkage. First the joint velocities of the linkage are graphically found of which the procedure is well known. To obtain the linear and angular momentum graphically, the mass and inertia of each element are modeled with two equivalent masses about the center of mass of the element, resulting in a mass and inertia equivalent model with solely point masses. The velocities of these point masses are obtained and each velocity vector is multiplied with the respective mass value to obtain vectors that represent the linear momentum. For force balance it is shown that the sum of all linear momentum vectors form a polygon. Subsequently the linear momentum vectors with their moment arms are transferred into an angular momentum diagram which for moment balance shows to sum up to zero.

Keywords: Graphic analysis · Linear and angular momentum · Gravity force balance · Shaking force and shaking moment balance · Pantograph.

1 Introduction

When mechanisms are shaking force and shaking moment balanced, they do not exert any dynamic reaction forces and moments to their base during (high-speed) motions [3,5]. This reduces base vibrations significantly and when placed on floating platforms such as drones and cable robot end-effectors, balanced mechanisms do not disturb the position, orientation, and motion of the floating platform [2,7]. For shaking force balance the linear momentum of all moving elements together must be constant (zero) while for shaking moment balance the angular momentum of all moving elements together must be constant (zero). A shaking force balanced mechanism is also gravity force balanced and therefore all methods for shaking force balancing are also applicable for gravity force balancing.

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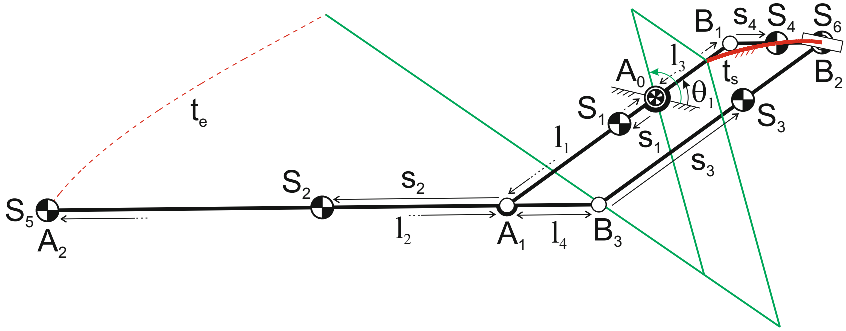

Fig. 1. 1-DoF dynamically balanced pantographic linkage in two poses with fixed joint A_0 and a slider in B_2 with slider trajectory t_s in red. The common CoM of all elements is stationary in A_0.

While there are various methods for force balancing of a linkage, using countermasses [1] or inherently balanced linkage architectures [6], for moment balancing of a linkage the methods are extremely limited [4]. Obtaining moment balance without additional elements is in most cases not possible [9].

In order to obtain a better understanding of moment balancing, the goal of this paper is to present a graphical method for the analysis of the linear and angular momentum of a linkage and to apply it to a 1-DoF dynamically balanced pantographic linkage to graphically show that the sums of linear and angular momenta are indeed zero. This will also give insight in the contribution of each linkage element to the total linear and angular momentum, i.e. in their contributions to the dynamic balance.

First the linkage is presented and the joint velocities are found graphically. Then the mass and inertia of each element are modeled with two equivalent masses to obtain a mass and inertia equivalent model with solely point masses. Subsequently the velocities and the linear momenta of the point masses are obtained graphically and are evaluated for force balance. As a final step, an angular momentum diagram is presented for evaluation of the moment balance.

2 Graphic Analysis of the Linear and Angular Momentum

Figure 1 shows a pantographic linkage consisting of the 4 links B_1A_1, B_3A_2, B_3B_2, and B_1B_2, which are connected with revolute pairs in A_1, B_1, B_2, and B_3, forming a parallelogram. In A_0 of link B_1A_1 the linkage has a revolute pair with the base, i.e. A_0 is the fixed joint. In joint B_2 there is a slider with fixed slider trajectory t_s which constrains the linkage to one degree-of-freedom (1-DoF) motion, indicated by angle θ_1 of the absolute rotation of link B_1A_1. When in motion, the endpoint A_2 traces the trajectory t_e. The linkage is shown for two poses, the extended pose at the beginning of the slider trajectory with a relative

Table 1. Parameter values for dynamic balance of the pantographic linkage in Fig. 1.

[mm]	[mm]	[g]	[gmm^2]
$l_1 = 100$	$s_1 = 24.36$	$m_1 = 118.56$	$I_1 = 312454$
$l_2 = 250$	$s_2 = 100.57$	$m_2 = 302.95$	$I_2 = 2482640$
$l_3 = 50$	$s_3 = 97.05$	$m_3 = 150.19$	$I_3 = 503989$
$l_4 = 50$	$s_4 = 25.48$	$m_4 = 24.22$	$I_4 = 7275$
		$m_5 = 67.16$	$I_5 = 4581$
		$m_6 = 782.63$	$I_6 = n/a$

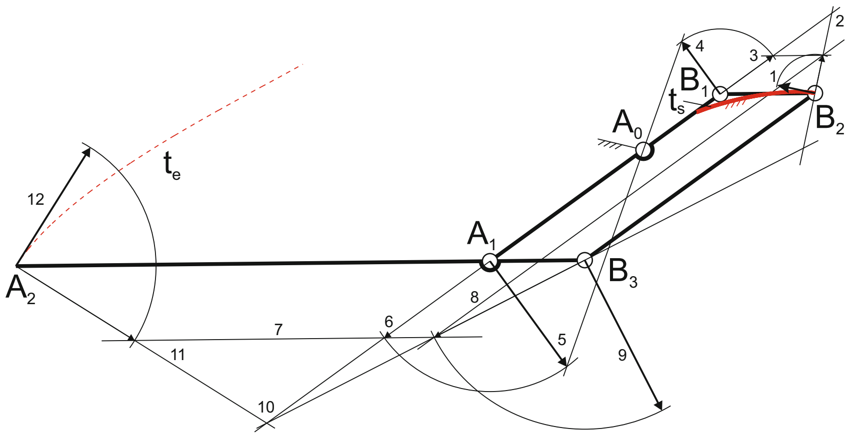

Fig. 2. Graphic analysis of the joint velocities with the velocity in B_2 known.

angle between links B_1A_1 and B_3A_2 of 35.4° and, in green, the retracted pose at the end of the slider trajectory.

The links have lengths l_i as illustrated in Fig. 1 and each of the 4 links has a mass m_i and an inertia I_i about the link center-of-mass (CoM) S_i, which is at a distance s_i from a joint as depicted. In addition, in A_2 there is the CoM S_5 of the end-effector with mass m_5 and inertia I_5, rigidly mounted on link B_3A_2, and in B_2 there is the CoM S_6 with mass m_6, which is the mass of the slider parts and a countermass together. The inertia of these last parts is not considered since they are solely in translational motion (the slider consists of a pin-in-slot of which the pin is fixed with and included in link B_3B_2 and the piston component that actuates the pin-in-slot motion is moving rectilinearly; the countermass is a circular disc on the slider pin with negligible friction in between for which it does not rotate). With the values in Table 1 the linkage is completely force balanced and moment balanced, which was verified by a dynamic simulation showing that during motion the common CoM of all elements remains stationary in A_0 and that the sum of the angular momenta of all elements remains zero.

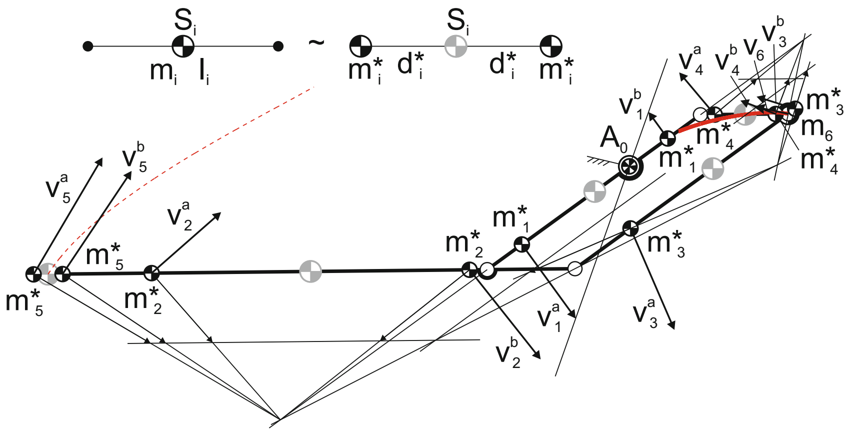

Fig. 3. Velocity analysis of the point masses of a dynamically equivalent model where the mass and inertia of each element are modeled with two equivalent masses.

The first step is the graphic analysis of the joint velocities, which is shown in Fig. 2 following the commonly known approach [8]. The velocity in B_2 is known as starting point and has a direction tangent to the slider trajectory t_s. By rotating it 90°, the intersection (2) with the line through A_1B_1 is found and subsequently intersection (3) and the velocity of B_1 (4) are derived. With the line through A_0 then the velocity of A_1 is obtained (5) which, after 90° rotation (6), determines line (7) which is parallel to line A_1A_2. The intersection of line (7) with line (8), which is parallel to line B_2B_3, determines the velocity of B_3 (9). Finally intersection (10) is obtained with which line (11) to A_2 is determined and the velocity in A_2 is found (12). As expected, the velocity vector in A_2 is indeed tangent to the traced end-effector trajectory t_e.

To be able to graphically analyze the angular momentum of the linkage, the mass and inertia of each element are modeled with two equivalent masses. This is the simplest possibility for dynamic equivalent modeling of planar motions for which also more than two equivalent masses can be used [10]. Figure 3 shows the dynamically equivalent model where each mass m_i, except m_6 for which no inertia is involved, has been divided in two equal equivalent masses $m_i^* = m_i/2$ both located at a distance d_i^* from the element CoM S_i, one on each side along the line through the link joints such that S_i is their common CoM. The distances d_i^* are determined by the inertia of the element and are derived from $I_i = 2(m_i^* d_i^{*2})$ as $d_i^* = \sqrt{I_i/m_i}$ with which the model is both mass and inertia equivalent with solely point masses. It is also possible to divide m_i in two different equivalent masses with two different lengths d_i^* or to place the equivalent masses off the line through the link joints which, however, would make the analysis more complicated than needed.

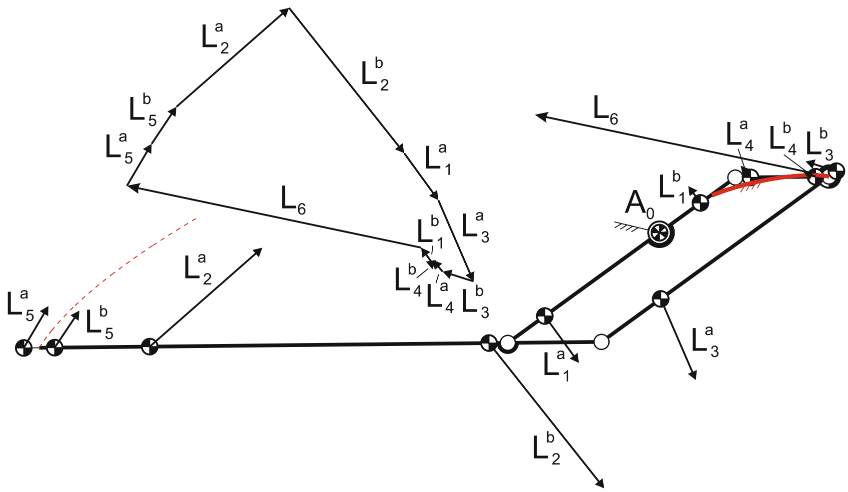

Fig. 4. Linear momentum vectors obtained by multiplying the velocity vectors with the respective mass values. The sum of the linear momentum vectors forms a polygon for force balance.

The velocity analysis of the 11 point masses is also shown in Fig. 3. Continuing with the graphical solution of the joint velocities in Fig. 2, with the instantaneous link centers of rotation the velocities of all point masses are readily obtained. This might contrast with the readability of the illustration of Fig. 3, for which the author apologizes.

The linear momentum of each point mass is obtained when each velocity vector in Fig. 3 is multiplied by its mass value. The resulting linear momentum vectors are shown in Fig. 4 which were obtained by multiplying the length of each velocity vector by its respective value $m_i/100$ with the mass values in Table 1, scaling the vectors to fit within the drawing.

The force balance can now be verified by adding all the linear momentum vectors together, which must form a polygon (i.e. a closed chain) since this means that the sum of the linear momenta of the linkage equals zero. The linear momentum polygon is also shown in Fig. 4.

The angular momentum of the linkage consists of the sum of the moments of the linear momentum vectors about the common CoM in A_0. The moments of the linear momentum vectors are illustrated in Fig. 5 where each linear momentum vector has been shifted along its line of action to the endpoint of its moment arm. The angular momentum diagram in Fig. 6 is obtained from Fig. 5 when all the linear momentum vectors are rotated such that their moment arms are aligned with the same line u. Then all the linear momentum vectors are oriented vertically, either upwards or downwards.

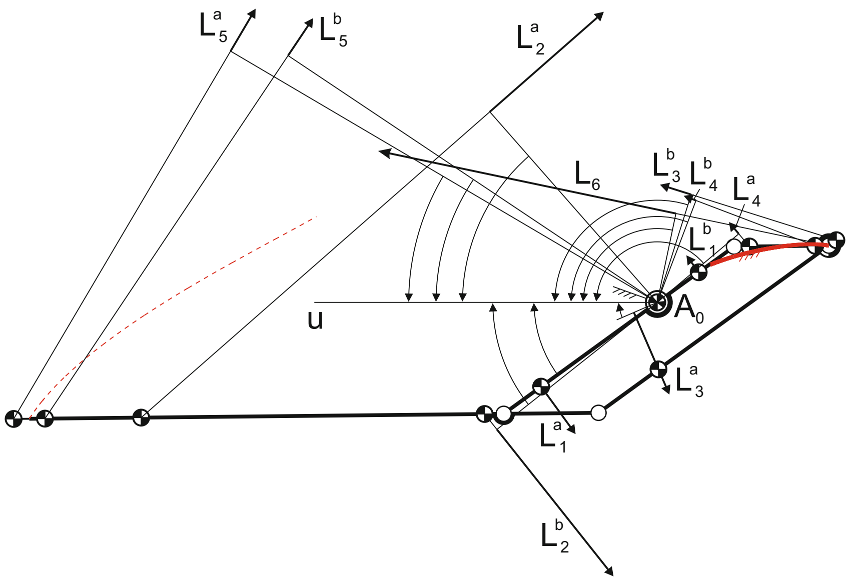

Fig. 5. Representation of the angular momenta of the linkage with the linear momentum vectors and their respective moment arms about the common CoM in A_0.

The angular momentum of each linear momentum vector can be found by the graphical multiplication of lines for which the shown triangle of reference is used with a height of L_6 and a reference width equal to the moment arm of L_5^a. This results for each linear momentum vector into a diagonal line, which starts at 0 at the location of the vector (i.e. at the end of the moment arm) and crosses the vertical line H through A_0 - the angular momentum axis - at the value of its angular momentum.

For example, when L_2^a is placed in the reference triangle at the location of L_6, which is shown in green, then the diagonal line of L_2^a is found as the line from the endpoint of L_2^a to the endpoint of the triangle. Subsequently this diagonal line is placed in the diagram at L_2^a on line u and crosses the H-axis in point h, which is the value of the angular momentum of L_2^a.

To sum the resulting angular momentum values, the diagonal lines have been vertically shifted such that each diagonal line starts at the height of the endpoint of the previous diagonal line. Of the upward directed linear momentum vectors the summed angular momentum is shown below A_0 and of the downward directed linear momentum vectors the summed angular momentum is shown above A_0. For the total sum of the angular momenta to be zero, the part above A_0 must be equal to the part below A_0, which is verified by the circular arc about A_0.

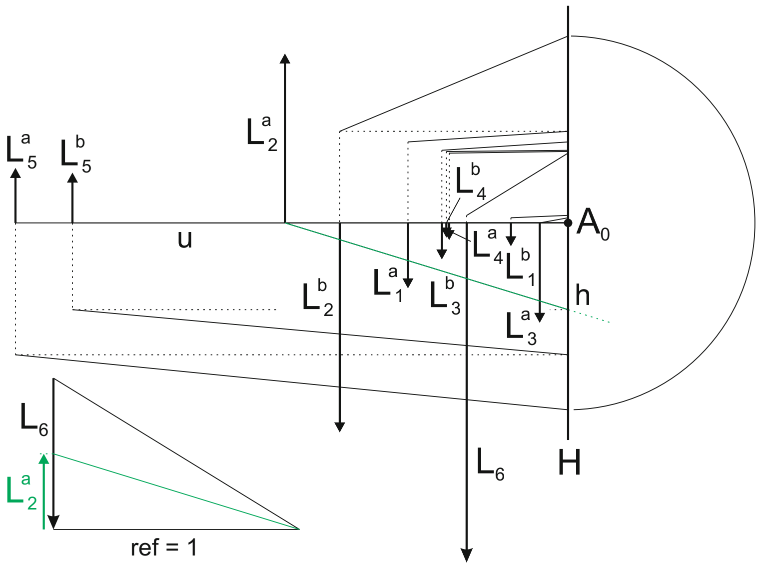

Fig. 6. Angular momentum diagram obtained by rotating all the linear momentum vectors such that their moment arms are aligned with the same line u. The angular momentum of each vector is found by the multiplication of lines with the help of a triangle of reference, gaining diagonal lines that intersect with the vertical angular momentum axis H through A_0 at the angular momentum values. The values of all intersections sum up to zero for moment balance.

3 Conclusions

In this paper it was shown how the linear momentum and the angular momentum of a linkage can be found graphically. As an example this was applied to verify the shaking force balance and the shaking moment balance of a 1-DoF pantographic linkage. The mass and inertia of each linkage element were modeled with two equivalent masses to obtain a dynamically equivalent model with solely point masses. The velocities of the point masses were derived graphically and the linear momenta of the point masses were found by multiplying the velocity vectors with their respective mass values. For force balance it was shown that the sum of the linear momentum vectors form a linear momentum polygon. The angular momentum was presented in an angular momentum diagram and showed to sum up to zero for moment balance. The presented graphical method may be of help to better understand the characteristics of force balance and, in specific, the characteristics of moment balance for the development of a synthesis method for moment balanced mechanisms. Extending the method to linkages with multiple degrees of freedom and to spatial linkages is an interesting next step.

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