2019 IEEE 7th International Conference on Control, Mechatronics and Automation

An Origami-Inspired Spherical Transformable Metamaterial Based on Symmetry Groups

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***Abstract*—Most currently existing mechanical metamaterial designs are based on Bravais lattices. These consist of parallelogram or parallelepiped unit cells, which are respectively translated along two or three independent vectors to fill the complete space. This approach is inherently unable to match curved surfaces like spheres, since these cannot be constructed only from parallel and perpendicular lines. In this paper, we introduce a generalized unit cell, based on the symmetry groups of the sphere. We use this approach to develop a spherical transformable origami-inspired metamaterial. We describe the motions of this new metamaterial, as well as experimental observations on a physical, 3D printed model.**

Keywords-mechanical metamaterials; origami; periodic mechanisms; spherical symmetries

I. INTRODUCTION

Mechanical metamaterials are an emerging class of materials where an internal structure, rather than the chemical structure of the material they are made of, dominates the bulk material properties [1], [2]. Generally, when designing these mechanical structures, a unit cell (UC) is designed, of which many copies are repeatedly placed along two or three lattice vectors, for planar and spatial metamaterials respectively, to fill all of space according to a Bravais lattice [3]. This approach greatly simplifies the design process, since it is assumed that the internal deformations of the material structure follow the same periodicity. In general, the mechanical behavior of the metamaterial is fully determined by the behavior of the single unit cell. Therefore, the mechanical behavior that is designed in a single unit cell determines the mechanical properties of the bulk.

While this approach lends itself well to create infinite areas or volumes, not all finite shapes can be created in this way. Specifically, when we consider curved surfaces, we are able to approximate their shape by a stacking of parallelepipeds, but will, in general, never be able to exactly match all curvatures. To illustrate this, consider building a sphere of LEGO bricks. With increasing numbers of available bricks, the sphere can be better approximated, but an exact matching will never be achieved, as illustrated in Fig. 1.

In this paper, we present a generalized way of constructing a unit cell, based on the symmetry groups of the desired shape, which is a sphere in our case. Using this method, the unit cell is not copied and translated along generators of a Bravais lattice, but the copies of the unit cell are rotated and mirrored according the chosen symmetry groups of the sphere. Because of this it will be shown that the whole sphere can be exactly matched by the designed mechanical structure.

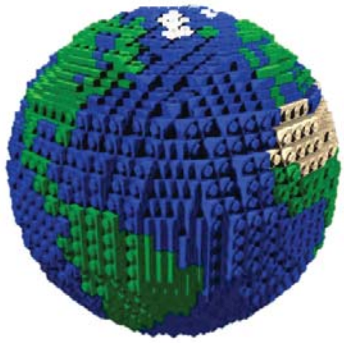

Figure 1. When creating a sphere out of LEGO bricks (or other parallelepipeds), only an approximation of the sphere can be achieved. *Adapted from* [4].

We employ this method to design a transformable origami inspired sphere, consisting of 60 generalized unit cells. These unit cells have a single degree of freedom, allowing the sphere to change in size, enabling a reduction of the inscribed volume of 76.3%.

We present a kinematic analysis of this mechanical structure as well as experimental validation of a 3D-printed prototype. We discuss the performance of the design as well as the number of degrees of freedom when periodicity is relaxed to the whole sphere.

II. SPHERICAL SYMMETRY GROUPS

For spheres, there are exactly 14 possible symmetry groups [5], [6]. These groups can be further split into two categories: platonic and parametric symmetries. Seven platonic symmetry groups exist, corresponding to the Platonic solids. There are seven parametric groups, for which the number of fundamental domains that are needed to span the whole sphere depends on a parameter.

In this study, we chose to focus on the Platonic groups, since for these groups the fundamental domains are more evenly distributed over the sphere due to their correspondence to regular polyhedra. For the parametric groups, there are two sets of fundamental domains, one on the “northside” of the sphere, and one on the “south-side”, leading to a less evenly distributed set of domains. Since, as we describe in the next section, a polyhedral approximation of the sphere will be used in our design, the even distribution of the fundamental domains leads to a better correspondence between the mechanism and the original sphere.

Within the Platonic groups, the *532 symmetry group (using the notation from [5]) was chosen, as it leads to the largest number of fundamental domains on the sphere. This group corresponds to the dodecahedron and its dual, the icosahedron. The fundamental domains of this symmetry group correspond to the faces of a dysdiakis triacontahedron, shown in Fig. 2. This polygon has 120 faces and is the largest Catalan solid.

III. UNIT CELL DESIGN

We design the origami-inspired spherical mechanism based on a unit cell consisting of rigid faces and folds. In general, the folding of curved surfaces is not possible without tearing or buckling. Therefore, to create a folding mechanism, we need to approximate the sphere with a polyhedron. The choice for *532 symmetry in the previous section leads to the dysdiakis triacontahedron.

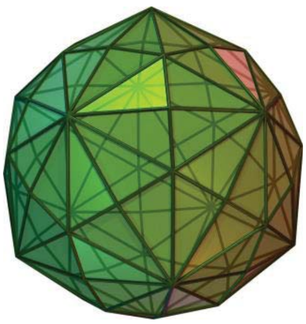

Figure 2. The fundamental domains of the spherical *532 symmetry group correspond to the faces of a dysdiakis triacontahedron.

Source: [7]

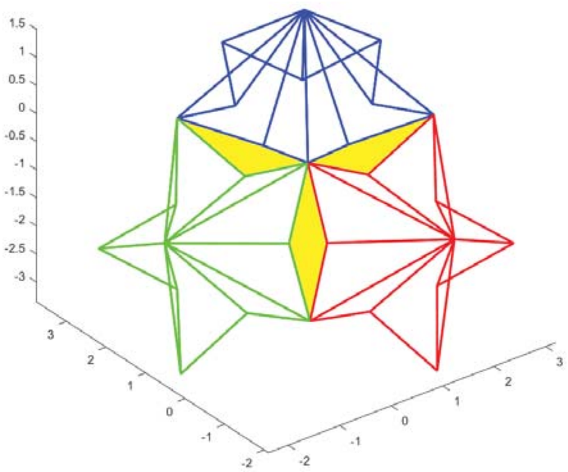

Figure 3. The origami-inspired structure consists of rigid triangular faces, which are connected by folds into pentagonal units (shown with red, blue and green outlines). These units are subsequently coupled by spherical joints, allowing for open spaces (shown in yellow) when the structure is deformed.

For our origami-inspired mechanism, we take all these faces to be rigid and design the connections between the faces such that a dilational motion is obtained. For this, we consider a Unit Cell consisting of two faces, connected by a fold. Five of these unit cells can be coupled, again with folds, to form a pentagonal unit. The connections between the pentagonal units are realized by spherical joints at the corners of the base pentagon. This structure, for three of such pentagonal units, is shown in Fig. 3.

When the unit cells are repeated all over the sphere in this way, twelve of these pentagonal units can be used to span the sphere, preserving the single degree of freedom of the unit cell. At the construction position, without open spaces between the pentagonal units, it is a dysdiakis triacontahedron, and when actuated, the sides of the pentagonal units move inwards, shrinking the side-lengths of the regular pentagon they describe. This creates open spaces between these pentagonal units, leaving only point contacts between them. In the physical model, these connections are realized by adding two plates to the unit cell, which are folded into the polyhedron in the construction position. In this way, the whole mechanism consists of rigid plates, connected only by fold lines.

IV. FABRICATION METHODS

Of the model shown in Fig. 3 a prototype was designed and 3D printed in parts from PLA, using a Prusa i3mk2 3D printer. The 12 printed parts were aligned using notched connectors at their sides and cyanoacrylate glue was used to fix these connections. The design of one such part is shown in Fig. 4.

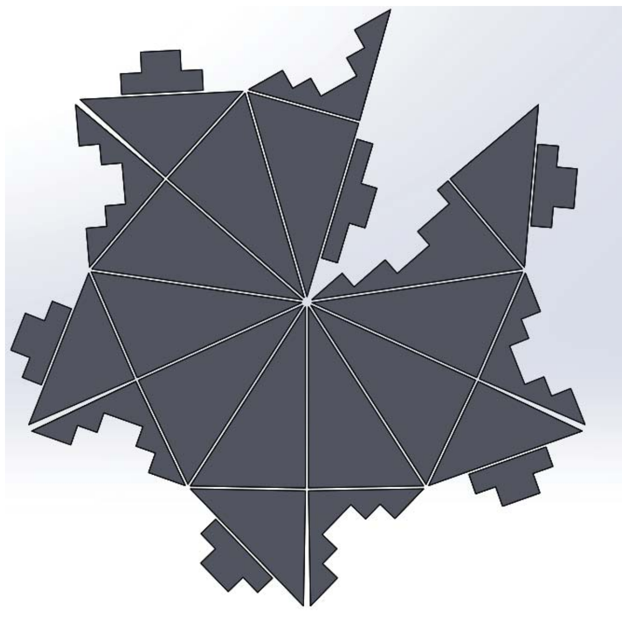

Figure 4. A top view of the pentagonal unit design. These structures were 3D printed in the shown flat state. Netting was used to create fold lines between the grey rigid plates and twelve of these units were assembled into a Dysdiakis Triacontahedron by matching and gluing the notched parts.

The parts were 3D printed in a flat state. Each part consisted of a single pentagonal pyramid, opened up at one of the fold lines to fold it flat. The total thickness of the plates is 3mm. For the fold lines, two layers of netting were included within the 3D print, one at a height of 0.5mm and one at a height of 2.75mm. These layers were included during printing and the following layers of PLA flows between the netting, securing it within the rigid plats. After 3D printing, the top- and bottom layers of netting were cut selectively to assign mountain- and valley folds to the mechanism. This resulted in an origami mechanism consisting of rigid plates, connected by fold lines with very little stiffness.

V. Results

A. Degrees of Freedom

The generalized unit cell we designed has a single degree of freedom. However, when the whole sphere is considered, instead of just the periodic motions based on a single unit cell, there are 33 degrees of freedom. This number was calculated using counting-arguments, and verified by modeling the whole structure as a pin-jointed framework and calculating the nullspace of the Jacobian [8].

In the physical model, these extra degrees of freedom made it difficult to uniquely control the mechanical structure. A balloon was inserted into the enclosed volume such that by inflating the balloon, the prototype expands symmetrically, as if it has a single degree of freedom. To investigate the practical volume decrease, the balloon was deflated and the structure was collapsed manually.

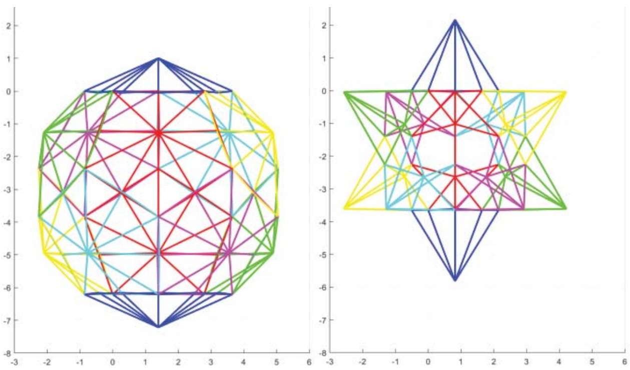

Figure 5. The expanded (left) and contracted (right) state of the origami inspired mechanism.

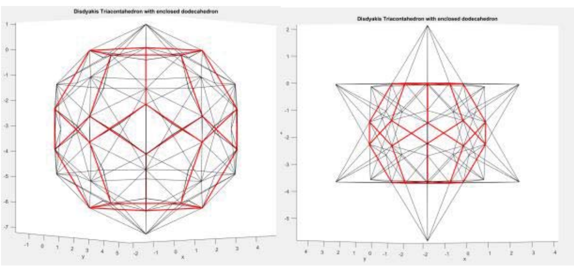

Figure 6. The volume change of the mechanism is determined based on the inscribed dodecahedron, defined by the connection points of the pentagonal units. This dodecahedron is shown here in red.

B. Volume Change

When the mechanism is contracted, the pentagonal units deform such that their base pentagon remains regular and the side-lengths decrease. Simultaneously, the pentagonal units become more pointed, creating a shape similar to a stellated dodecahedron. This state is shown, next to the fully expanded state, in Fig. 5.

To determine the volume decrease of the kinematic model under actuation, the inscribed dodecahedron, defined by the connection points of the pentagonal units was used. This is illustrated in Fig. 6. This volume was calculated in the kinematic model to undergo a decrease of 76.3%.

Images of the physical model in both the expanded and contracted state are shown in Fig. 7. In this case, the volume decrease achieved was less than the calculated value. This is clearly shown in Fig. 8, where the two states are superimposed in one figure. The enveloping sphere of the model hardly changes in size.

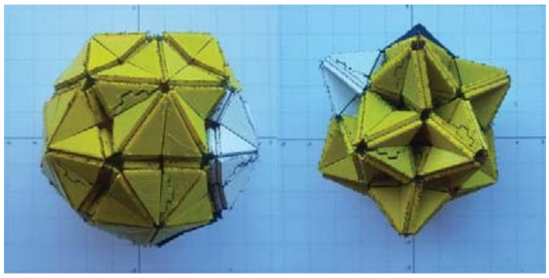

Figure 7. The physical model in the expanded state (left) and in the contracted state (right).

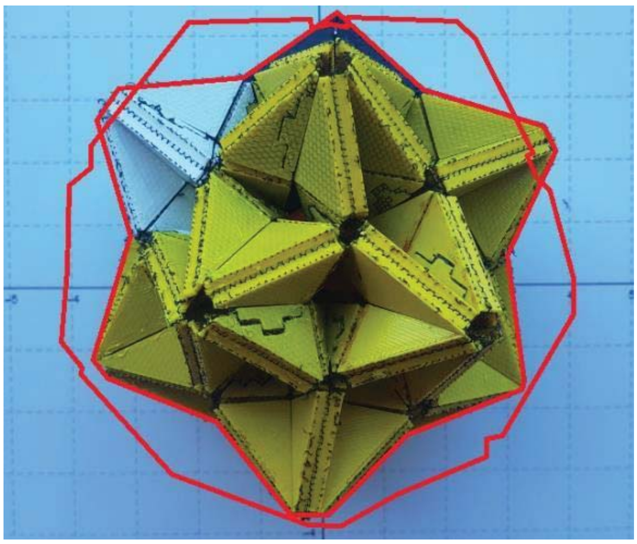

Figure 8. Comparison of the expanded and contracted state of the physical model. The contracted state is shown, with the contours of both the contracted and the expanded state drawn in red.

VI. Discussion

While the volume decrease of the kinematic model is not matched by the physical model, the deformed shapes match. The differences between the two are likely caused by the thickness of the plates in the physical model. This was assumed to be zero in the kinematic model, while they physically limit the motion of the prototype. Also, because the physical model had 33 degrees of freedom and was actuated by hand, it is likely that the extremal states of the mechanism were not reached. For example, the expanded state shown in Fig. 7 shows small indentations on the edges of the pentagonal units, indicating that the fully expanded state was not reached.

A balloon was used to expand the structure, but the contraction was done manually in this study. As an improvement, the balloon could be attached to the inside of the mechanism, also allowing control of the mechanism contraction through deflation of the balloon. Alternatively, a different assignment of fold lines might help to reduce the degrees of freedom.

Finally, the mechanism presented in this paper deforms from a dysdiakis triacontahedron into a stellated dodecahedron. If the largest diameter of the structure is considered, the resulting scaling is minimal. An alternative design could have the pentagonal units pointed into the enclosed volume, creating an approximate dodecahedron with dimples in the faces, as illustrated in Fig. 9. This design would have a greater volume change of the circumscribed dodecahedron, but the internal collisions would not allow the neighboring faces to fold completely flat.

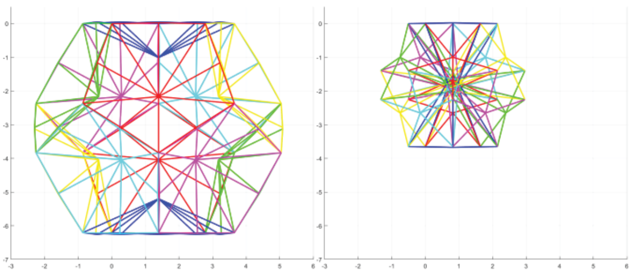

Figure 9. An alternative design, where the pentagonal units are inverted to point into the enclosed volume, for improved scaling of the circumscribed volume. Shown are both the expanded (left) and the contracted state (right).

VII. Conclusions

In this paper, we have presented an origami-inspired transformable mechanism whose construction was based on the spherical *532 symmetry group. The sphere is designed to perform a dilational motion, which results in a decrease of the inscribed dodecahedron volume of 76.3%.

In order to construct this spherical structure, we developed a generalization of the periodic lattices regularly used for the design of mechanical metamaterials. The generalized unit cell consists of a fundamental domain of the chosen spherical symmetry group, and instead of building the lattice by discrete translation, the corresponding symmetry operations are used. The resulting design was constructed using additive manufacturing, with netting used to obtain low stiffness fold lines between the rigid plates.

Acknowledgment

This work was supported by the Nano Engineering Research Initiative of Delft University of Technology (www.tudelft.nl/neri).

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